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COLLECTIVE COORDINATE QUANTIZATION OF DIRICHLET BRANES*

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Abstract

Collective coordinate quantization of Dirichlet branes is discussed. Utilizing Polchinski's combinatoric rule, semiclassical D-brane wave functional is given in proper-time formalism. D-brane equation of motion is then identified with renormalization group equation of defining Dirichlet open string theory. Quantum mechanical size of macroscopically charged D-brane is illustrated and striking difference from ordinary field theory BPS particle is emphasized.

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I. INTRODUCTION

Recently nonperturbative string theory has given us many surprising results. There are now compiling evidences that all known perturbatively defined string theories are related each other by duality at nonperturbative level [1]. Central to this advance was progress to semiclassical string theory, in particular, deeper understanding of stringy topological solitons over the last few years [2–7]. Previous study of string solitons, however, has been restricted mainly to low-energy effective field theory approximation. While exact conformal field theories in a few cases has been known from the earliest days [3,8], further progress was hampered because of technical difficulties in dealing with geodesic motion interpolating between different conformal field theories, viz. space of nontrivial vacua of string field theory.

In a recent remarkable work [9], Polchinski has obtained an exact conformal field theory describing Ramond-Ramond charged solitons. These so-called D(irichlet)-branes are described in terms of Dirichlet open strings that are coupled to the underlying type II closed strings. Polchinski's work has cleared up many puzzling aspects that arose previously when string solitons were studied within low-energy effective field theory truncation. The D-brane proposal has already passed many nontrivial consistency tests but all of them so far were mainly on static properties. With its simplicity and exactness it should now be possible to study *quantum dynamics* of string solitons in detail.

In this talk I report my recent work [10] on several aspects of D-brane dynamics: collective coordinates, semi-classical quantization, renormalization group interpretation of equation of motion and quantum mechanical size of macroscopically charged D-brane.

II. D-BRANE AND COLLECTIVE COORDINATES

Consider a conformally invariant two-dimensional system. If a boundary is introduced at which the bulk system ends, then it is well-known that microscopic detail is renormalized into a set of conformally invariant boundary conditions. For a Gaussian model such boundary

conditions are either Neumann or Dirichlet boundary conditions but not a combination of the two.

Similarly, in closed string theory, it is possible to introduce worldsheet boundaries. At each boundaries, string coordinates $X^\mu(z, \overline{z})$ may be assigned to either Neumann (N) or Dirichlet (D) boundary conditions. Mixed boundary condition may seem break 10- or 26-dimensional Lorentz invariance. However, on toroidally compactified spacetime, target space duality $R \leftrightarrow \alpha'/R$ interchanges N and D boundary conditions. Hence Lorentz invariance is maintained up to target space T-duality. Denote N-coordinates as $X^i, i = 0, 1, \cdots, p$ and D-coordinates as $Y^a, a = p+1, \cdots, 9(25)$. Each worldsheet boundaries are mapped into spacetime *extrinsic* hypersurfaces of dimension $(p+1)$ spanned by X^i, viz. Dirichlet p-brane world-volume. Polchinski [9] has shown these D-branes are nonperturbative states of type II strings that carry RR-charges obeying Dirac quantization condition and that saturate BPS bounds.

Worldsheet chiral symmetries restrict possible p-branes further. Type IIB strings are worldsheet symmetric that even numbers of D-coordinates are possible, hence, contains $p = -1, 1, 3$ branes and their magnetic duals. Similarly, for type IIA odd numbers of D-coordinates are allowed, viz. $p = 0, 2, 4$ branes and their magnetic duals. Since IIA and IIB string theories are mapped into each other under target space duality $R \rightarrow \alpha'/R$, one can build up all D-branes from the oriented open string sector ($p = 9$) in IIB theory and cascade T-duality transformations. In type I string theory, because of worldsheet orbifold projection, only $p = 1, 5, 9$ branes are allowed.

Worldsheet interaction of type II strings with D-branes are described by Dirichlet open string theory. Worldsheet interaction at each boundaries is deduced by cascade T-duality transformation of the known oriented 9-brane (open string) theory. For massless excitations, the worldsheet interactions at each boundaries are described by

$$S_B = \oint d\tau \Big[\sum_{i=0}^{p} A_i(X^0, \cdots, X^p) \partial_t X^i + \sum_{a=p+1}^{9(25)} \phi_a(X^0, \cdots, X^p) \partial_n Y^a \Big] \tag{1}$$

Image of 'Chan-Paton quark' (end of Dirichlet open string) is mapped onto D-brane

world-volume Σ_{p+1}, but otherwise can move freely on it by $(p+1)$-dimensional translation invariance. Such restriction is consistent if gapless gauge field excitation described by the vertex operator $V_A = \oint A_i \partial_t X^i$ is present only on Σ_{p+1} hypersurface but not outside, hence, $A_i = A_i(X^j)$. This world-volume gauge fields also mix with type II Kalb-Ramond field $B_{\mu\nu}$ as is evident from bulk-extended expression of the vertex operator $V_A = i \int d^2 z \partial_z (A_i \partial_{\overline{z}} X^i) - (c.c.)$. This is the well-known Cremmer-Scherk [11] coupling and generates stringy Higgs mechanism. The coupling also makes it clear what the meaning of world-volume gauge field is: closed string winding modes transmutes into massive world-volume gauge fields and provides $SL(2, \mathbf{Z})$ orbits of Neveu-Schwarz and Ramond charges to the D-brane BPS mass.

The vertex operator $V_\phi = \oint \phi_a(X^i) \partial_n Y^a$ describes transverse translation of local D-brane world-volume X^i, hence, *collective coordinates* . Normally spacetime translations are redundant and correspond to null states. This is clear from rewriting $V_\phi = \int d^2 z \partial_z (\phi_a \partial_{\overline{z}} Y^a) +$ $(c.c.)$, which decouples on a compact worldsheet. The decoupling fails precisely when D-boundaries are present and V_a's turn into genuine physical modes. This is consistent with spacetime point of view since, in the presence of a D-brane, translational symmetry is spontaneously broken and new Goldstone mode states should appear. The V_a's that fail to decouple and fail to be null are precisely those Goldstone states.

Low-energy spacetime interactions type II strings with N independent D-branes are then described by massless modes of Dirichlet string theory: $D = 10, N = 2B$ supergravity coupled to $D = 10, N = 1$ Dirac-Born-Infeld $U(N)$ gauge theory on Σ_{p+1} dimensionally reduced and T-dualized onto Σ_{p+1}. Mismatch of spacetime supersymmetry and $U(N)$ Chan-Paton gauge group does not cause problems since the D-brane excitations are confined only on Σ_{p+1}. Thus, dimensionally reducing and T-dualizing first and then making a 'nonrelativistic expansion' for small gauge and Goldstone field excitations [12],

$$\begin{aligned} S_{\rm wv} &= -{\rm Tr} T_p \int_{\Sigma_{p+1}} e^{-\phi} \sqrt{\det(G_{MN} + B_{MN} + F_{MN})} \\ &\rightarrow {\rm Tr} T_p \int_{\Sigma_{p+1}} e^{-\phi} \sqrt{G} \Big(1 + \frac{1}{4} (F + B)^2_{ij} + [D_i, \phi^a]^2 + [\phi^a, \phi^b]^2 \Big). \end{aligned} \tag{2}$$

Here $T_p e^{-\phi}$ denotes p-brane static mass density obtained from T-dual transform of 9-brane dilaton tadpole amplitude on a disk [13]. The first and second terms give bare energy and Casimir energy of static D-branes. The second term also contains aforementioned Cremmer-Scherk coupling, hence, manifest gauge invariance of Kalb-Ramond $B_{\mu\nu}$ field is maintained. The third and fourth terms are kinetic and potential energy for moving D-branes.

III. COMBINATORICS OF PERTURBATIVE DIRICHLET STRING THEORY

Consider string S-matrix elements involving D-branes. Dirichlet string theory associates each D-brane to a Dirichlet boundary on which lives an independent 'Chan-Paton quark'. Interaction between D-branes and string states are then described by Riemann surfaces with handles and holes. This also implies that apparently disconnected worldsheet diagrams are in fact connected in spacetime as long as boundaries from disconnected worldsheet are mapped into common D-brane(s). This entails a new rule of string perturbation expansions for the S-matrix generating functional Z [14]. Let $n = 1, \cdots, N$ label N independent D-branes into which different species of 'Chan-Paton quarks' are mapped. Worldsheet perturbation theory for Z is then organized as

$$Z_{\rm new} = \sum_{N=0}^{\infty} \frac{1}{N!} \otimes \Big(\prod_{n=1}^{N} \int [dY_n^a] \Big) \otimes \sum_{h=0}^{\infty} \frac{1}{h!} \otimes \sum_{a_1, \cdots, a_h = 1}^{N} \frac{h!}{m_1! m_2! \cdots m_N!} \tag{3}$$

where $m_i \geq 0, \sum_i m_i = h$. For fixed N, we sum over the number h of worldsheet boundaries and sum each of the h 'Chan-Paton quarks' (independent D-branes) from 1 to N. Then we integrate over the transverse positions of each D-branes and finally sum over the total number N of D-branes. Since summing over the number of holes and the 'Chan-Paton quarks' amounts to summing over the number of holes mapped into a given D-brane, the above combinatorics can be organized as

$$\begin{aligned} Z_{\rm new} &= \sum_{N=0}^{\infty} \otimes \frac{1}{N!} \int \prod_{n=1}^{N} [dY_n^a] \otimes \sum_{h_1=0}^{\infty} \frac{1}{h_1!} \sum_{h_2=0}^{\infty} \frac{1}{h_2!} \cdots \sum_{h_N=0}^{\infty} \frac{1}{h_N!} \\ &= \sum_{N=0}^{\infty} \otimes \frac{1}{N!} \prod_{n=1}^{N} \Big(\int [dY_n^a] \sum_{h_n=0}^{\infty} \frac{1}{h_n!} \Big) \end{aligned} \tag{4}$$

Exponentiation of disconnected worldsheets then generate a complete S-matrix generating functional Z.

In old perturbation theory [15] each Dirichlet boundaries are mapped into independent spacetime points, subsequently integrated over

$$Z_{\rm old} = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{n=1}^{N} \int [dY_n^a] \sum_{h_n=0}^{\infty} \frac{1}{n_a!} \delta_{n_a,1} \tag{5}$$

The difference arises because the D-branes are extrinsic structure to spacetime.

Combinatorics for the S-matrix generating functional in Dirichlet string perturbation theory may be rephrased as follows. Prepare for m disconnected, compact Riemann surfaces, create n_h holes arbitrarily distributed among the m Riemann surfaces. Map each holes to the world-volume of N independent D-branes allowing duplications. Finally sum over m, n_h, N independently with appropriate combinatoric factors $\mathcal{S}_{\rm ms}$ of n_h boundaries into m Riemann surfaces and $\mathcal{S}_{\rm st}$ of n_h boundaries into N D-branes

$$Z_{\rm new} = \sum_{n_h=0}^{\infty} \frac{1}{n_h!} \otimes \Big(\sum_{N=0}^{\infty} \frac{1}{N!} \mathcal{S}_{\rm st}(N \leftarrow n_h) \otimes \sum_{m=0}^{\infty} \frac{1}{m!} \mathcal{S}_{\rm ws}(n_h \rightarrow m) \Big). \tag{6}$$

We note that a single exponentiation maps each Dirichlet boundaries doubly into disconnected Riemann surfaces and into independent D-brane world-volumes in a symmetric manner.

IV. SEMI-CLASSICAL WAVE FUNCTION OF D-BRANES

Consider Dirichlet string partition function in the background of the type II string fields in which all worldsheet boundaries are mapped into a single D-brane world-volume Σ_{p+1}. The partition function serves as a generating functional, hence, S-matrix elements between D-brane and string states are derived from local variation of background string fields. The partition function is also related to the (Euclidean) wave functional Ψ_1 of the D-brane. The new combinatoric rule relates the wave functional to the partition function

$$Z_1 = \int [dY^a] \Psi_1[Y^a(\cdot)] \qquad \Psi_1[Y^a] = e^{\mathcal{S}_1}. \tag{7}$$

Here $\mathcal{S}_1$ sums up all one-particle irreducible *connected* worldsheet diagrams, whose boundaries are mapped to the D-brane world-volume. Integration over world-volume gauge field is already made for Ψ_1 to ensure type II winding quantum number conservation. Dirichlet string perturbation theory yields

$$\mathcal{S}_1[Y^a] = \sum_{h=1}^{\infty} e^{\phi(h-2)} S_{(h)} \tag{8}$$

in which $S_{(h)}$ denotes amplitude with h-holes. Sum over handles is implicitly assumed in the definition of $S_{(h)}$.

Higher order contributions $S_{(h\geq 2)}$ come from annulus, torus with a hole etc or sphere with three holes etc. They amount to D-brane mass renormalization. For type II string all except the disk diagram ($h = 1$) vanishes identically because of spacetime supersymmetry nonrenormalization theorem. For D-instanton, this is consistent with known results that the RR instantons are exact to all orders in string perturbation theory. The disk amplitude $S_{(1)}$ for type II superstring is easily obtained from 9-brane boundary state [16] after appropriate T-duality transformations. For simplicity, keeping only the transverse fluctuation of the D-brane world-volume

$$\begin{aligned}\mathcal{S}_1 &= T_p \int d\Sigma_{p+1} e^{-\phi} \sqrt{\mathrm{det} G_{ij}} \\ &= \int d\Sigma_{p+1} \Big[\frac{1}{V} \mathrm{det} G_{ij} - M_p^2 V \Big]\end{aligned} \tag{9}$$

where $M_p = T_p e^{-\phi}$. In the last expression, we have also introduced non-dynamical 'proper-time' variable V. Functional integral over V introduces no new Jacobian and amounts to a sum over all possible propagation of D-brane.

Consistency of Dirichlet string coupled to type II string requires to maintin conformal invariance or BRST invariance. The wave functional, however, contains various sources of logarithmic divergences that violate the conformal or BRST invariance. We have seen earlier that the Goldstone mode vertex operators are *isolated* , descendent operators that fail to decouple and to be null in the presence of the Dirichlet boundaries. It is now necessary to examine carefully all possible boundaries of moduli space. Crucial understanding on how to

do this has been made in a recent important work by Fischler *et.al.* [17]. Consider a finite but large proper-time interval T so that all scattering states of the D-brane form a discrete set of L_0 and $\overline{L}_0$ spectrum separated by a gap from the continuum of type II string states. With this infrared regularization provided we can properly extract divergent amplitudes unambiguously.

Consider the disk amplitude near a boundaries of moduli space for two sets of closed string vertex operators. Inserting a complete set of states labelled by $\{a\}$ that includes those naively BRST null and denoting propagators as Π_a, the disk amplitude

$$\begin{aligned}\langle\cdots\rangle_{D_2} \rightarrow & \sum_{\rm states} \langle\cdots|a\rangle_{D_2}\Pi_a(k)\langle a|\cdots\rangle_{D_2} \\ = & \int \frac{d^Dk}{(2\pi)^D}\,\langle\cdots V_A(k)\rangle_{D_2}\langle V_A^\dagger(k)\cdots\rangle_{D_2} \times \int_0^\infty \frac{dt}{t} e^{-t(k_n^2+m_n^2)} \\ + & \sum_{a=p+1}^{D-1} \langle\cdots V_\phi\rangle_{D_2}\langle V_\phi^\dagger\cdots\rangle_{D_2} \times \int_\epsilon^\infty \frac{dt}{t}.\end{aligned} \tag{10}$$

The first comes from physical excitations with continuum distribution labelled by momenta k_a, hence, does not cause any infrared divergence. The second is due to intermediate exchange of the D-brane Goldstone mode. Spacetime picture is that a tiny Dirichlet open string state propagates for a long proper-time and diverges linearly. As the D-brane Goldstone mode spectrum is discrete and isolated for a finite proper-time interval T, it is not possible to analytically continue kinematics and avoid infrared divergence as the cutoff $\epsilon \rightarrow 0$. It is precisely these divergences we need to cure.

Similarly the annulus amplitude near a boundary of moduli space at which the annulus strip is pinched into a thin, long open string propagation. While vanishing for stationary D-brane (BPS static force balance condition), the annulus amplitude with time varying D-brane velocity and/or with a background to soak up all the spacetime fermion zero modes are nonvanishing. Such amplitude also contains divergences

$$\begin{aligned}\langle\cdots\rangle_{A_2} \rightarrow & \sum_{\rm states} \langle a|\cdots|a\rangle\Pi_a(k) \\ = & \int \frac{d^Dk}{(2\pi)^D}\langle\cdots V_A(k)V_A^\dagger(k)\cdots\rangle_{D_2} \times \int_0^\infty \frac{dt}{t} e^{-t(k_n^2+m_n^2)}\end{aligned}$$

$$+\sum_{a=p+1}^{D-1}\langle\cdots V_{\phi_a}V^\dagger_{\phi_a}\cdots\rangle_{D_2}\times\int_\epsilon^\infty\frac{dt}{t}. \tag{11}$$

This diagram contains also infrared divergence due to D-brane Goldstone-mode exchange. Again spacetime picture is that a tiny Dirichlet open string propagates for a long proper-time interval and diverges linearly.

Noting that $V_{\phi_a}=\oint\phi_a\partial_n Y^a=\phi_a\partial/\partial Y^a$ viz. rigid translation of D-brane by ϕ_a transversally we find the two logarithmically divergent contributions combine into a total derivative

$$(\langle..\rangle_{D_2}+\langle..\rangle_{A_2})_{\log\epsilon}=(\log\epsilon)\Big[\frac{1}{2!}\phi\cdot\nabla_Y\langle..\rangle_{D_2}\phi\cdot\nabla_Y\langle..\rangle_{D_2}+\langle..\rangle_{D_2}(\phi\cdot\nabla_Y)^2\langle..\rangle_{D_2}\Big], \tag{12}$$

hence, to this order in e^ϕ, D-brane wave functional $\Psi_1[Y]$ contains

$$(\exp[\langle..\rangle_{D_2}+\langle..\rangle_{A_2}])_{\log\epsilon}\approx(\log\epsilon)\,\frac{1}{2!}(\phi\cdot\nabla_Y)^2\exp[\langle\cdots\rangle_{D_2}+\langle..\rangle_{A_2}]. \tag{13}$$

We have isolated leading $\log\epsilon$ divergences due to *worldsheet* short-distance singularity in the presence of Dirichlet boundaries. In *spacetime*, the divergence arises from propagation of isolated D-brane collective coordinate modes as the proper-time interval $T\to\infty$. Hence, the two regulators may be identified as $T\approx-\log\epsilon$ up to multiplicative and additive constants that can be determined from explicit S-matrix element calculations [10]. Logarithmic relation between the two should be evident if one recalls proper-time formalism [18] of Polyakov path integral: dilatation of worldsheet coordinates $\epsilon\to e^l\epsilon$ corresponds to shift of proper-time $T\to T-\log l$.

V. D-BRANE EQUATION OF MOTION AND RENORMALIZATION GROUP FLOW

Having isolated divergences in the presence of D-brane, how do we cure of them? Governing principle of string theory is the requirement of conformal or BRST invariance. Much the way spacetime equations of motion of string background fields has been obtained, the requirement applied to the Dirichlet string theory is expected to a new equation for consistent D-brane dynamics. With this motivation we now require

$$\epsilon \frac{d}{d\epsilon} Z_1 = \epsilon \frac{d}{d\epsilon} \Big(\int [dY^a] \Psi_1 [Y^a] \Big) = 0. \tag{14}$$

Since Z_1 and Ψ_1 sums up worldsheet diagrams of arbitrary number of handles and holes, Eq.(14) invokes Fischler-Susskind [19] mechanism in an essential way.

There are two possible ways to achieve this requirement. Noting that Eq.(13) is a total derivative with respect to the zero modes Y^a's, the first is to require that *integral* of Ψ_1 , viz. Z_1 itself satisfies conformal invariance requirement. This viewpoint has been advocated by Polchinski [9]: logarithmically divergent part Eq.(13) is a total derivative in Y^a-space and drops out upon integration over Y^a's in Z_1. Obviously in cases we are interested in *local* dynamics of D-brane this requirement does not offer much information. For example, given the semi-classical wave functional $\Psi[Y^a]$ first, how do we uncover an equation of motion to which the wave functional satisfies? The second viewpoint is then that the *integrand* of the path integral Z_1, viz. Ψ_1 is free from infrared divergence. Adopting this view point we get

$$\epsilon \frac{d}{d\epsilon} \Psi_1 [Y^a] = \frac{\phi^2}{2!} (\nabla_Y)^2 \Psi_1 [Y^a]. \tag{15}$$

Recalling that worldsheet variable $\log \epsilon$ is linearly to the spacetime proper-time interval T, the equation looks strikingly simiar to the Euclideanized Schrödinger equation. To show that this is not a mere coincidence, let us go back to the procedure of isolating logarithmic divergences in disk and annulus amplitudes. Conformal invariance requirement to $\Psi_1[Y]$ amounts to Wilson renormalization group equation for Dirichlet boundary action

$$\epsilon \frac{\partial}{\partial \epsilon} \Psi_1 [Y] = \frac{1}{2!} \oint d\tau_1 \oint d\tau_2 \epsilon \partial_\epsilon G_{ab}(t_1, t_2) \frac{\partial}{\partial Y^a(t_1)} \frac{\partial}{\partial Y^b(t_2)} \Psi_1 [Y]. \tag{16}$$

An important point is that the Dirichlet boundary Green function $G_{ab}(t)$ for transverse coordinates contains zero-mode part

$$G^{ab}(t_1, t_2) = \langle : Y^a(t_1) Y^b(t_2) : \rangle + (-\log \epsilon) |\overline{Y}^a|^2 \delta^{ab}. \tag{17}$$

The zero-mode $\overline{Y}^a$ of the transverse coordinates is a direct reflection of the spacetime zero-modes associated with the D-brane recoil. The zero mode is independent of the Dirichlet

string worldsheet variables and is proportional to $\log \epsilon$. Earlier identification that $-\log \epsilon \approx T$ also supports the interpretaion. The zero-mode is precisely the new source of conformal and BRST anomalies we have explicitly isolated in the previous section Eqs.(11) - (13).

So far we have examined the single logarithmic divergences and ways of ensuring their cancellation. Multiple logarithms are similarly cancelled as has been explicitly shown up to double logarithms for 9-branes [20]. The leading logarithms may be resummed and exponentiated to a new wave functional

$$\Psi_V[Y^a] = e^{(-\log \epsilon)\frac{\nabla_Y^2}{2M_p}} \Psi[Y^a]. \tag{18}$$

Physical meaning of this is as follows: recoiling D-brane acquires transverse kinetic energy $P^2/2M_p = M_p V^2/2$. During time interval $T \approx -\log \epsilon$, the (Euclidean) wave function acquires an additional phase (action) proportional to the kinetic energy. The leading log resummation is necessary since the kinetic energy is of the same order as the static energy $\mathcal{O}(1/\lambda)$ even though suppressed by velocity-squared. Since the new wave functional Ψ_V describes consistently a boosted D-brane, conformal or BRST invariance implies

$$\Big(\frac{d}{dT} + \frac{\nabla_Y^2}{2M_p}\Big)\Psi_V[Y^a] = 0. \tag{19}$$

Wilson renormalization group equation has been previously proposed [21] as a defining principle for obtaining string field equations of motion. The idea hs been extended to take into account of the Fischler-Susskind mechanism [22]. When applied to Dirichlet string theory we now see that consistent *D-brane equation of motion* Eq.(19) follows from the renormalization group equation. Equivalently, the equation can be understood as a consequence of on-shell Ward identity of type II string in BRST formulation [23]. Type II string contains BRST invariant conserved charges associated with translational invariance. The Dirichlet string boundary action Eq.(1) added to the type II string is naively BRST exact perturbations but fails precisely in the presence of D-brane. This means that the boundary action is a total BRST derivative of 'bad' operators that fail to decouple. Ward identities of spontaneously broken translational symmetry is then realized through non-decoupling of these 'bad' opearators and gives rise to a 'quantum master equation' similar to Eq.(15).

Full consideration of D-brane dynamics may require more careful analysis of dynamical gravity effect on the embedded D-brane world-volume. Previous experience with noncritical string theory [24,25] hints renormalization group flow equation changes time derivative in Eqs.(15,16,19) from first- to second-order

$$
\begin{aligned}
&[\partial_T^2 - 2M_p\partial_T - \nabla_Y^2]\Psi_V[Y^a] = 0 \\
\rightarrow\ &[\partial_T^2 - \nabla_Y^2 - M_p^2]e^{-M_pT}\Psi_V[Y^a] = 0,
\end{aligned}
\tag{20}
$$

viz., a covariant equation of motion for D-brane emerges. Similarly, massive Dirichlet string exchange is expected to generate contact interactions among D-branes and gives rise to nonlinear equation of motion [25].

VI. QUANTUM ASPECTS OF MACROSCOPICALLY CHARGED D-BRANE

So far I have discussed exclusively one-body aspects of D-brane. I now turn to a many-body aspects of macroscopically charged D-brane. Low-energy excitation of N overlapping D-branes is described by [12] dimensionally reduced $D = 10$ supersymmetric $U(N)$ Yang-Mills theory on Σ_{p+1}

$$S_{\rm wv} = {\rm Tr} T_p e^{-\phi} \int_{\Sigma_{p+1}} \sqrt{G}[\frac{1}{4}F_{ij}^2 + (D_i\phi^a)^2 + \cdots] \tag{21}$$

Essential many body-aspects is already present for type IIA D-particles , so I consider this case first. For macroscopically charged D-particles $N \rightarrow \infty$, world-line action is $U(N)$ *matrix* supersymmetric quantum mechanics. Gauge potential A_0 is nondynamical and but constrains the D-particle displacement $\phi^a(t)$ to a gauge singlet configuration. Diagonalizing the D-particle displacements ϕ^a

$$\phi^a = U{\rm diag}(y^1(t), y^2(t), \cdots, y^N(t))U^\dagger, \tag{22}$$

low-energy excitation is governed by an effective Hamiltonian

$$H_{\rm D-BPS} = {\rm Tr} \sum_{A=1}^{8} \{Q_A, Q_A^\dagger\} = -\frac{1}{2M_0}\sum_{a=1}^{N}\nabla_a^2 + \frac{1}{2}\sum_{a\neq b}^{N}\log(y_a - y_b) + \sum_{a\neq b}^{N}|y^a - y^b| + \cdots \tag{23}$$

The second term is quantum effective potential coming from functional integral after the diagonalization Eq.(22).

I now compare this with an effective Hamiltonian of macroscopically charged, field theory BPS particles. For the simplest BPS particles such as kinks in one dimension, low-energy dynamics is described entirely in terms of position of each particles. Hence, locally in the N-particle moduli space, effective action is given by N-dimensional *vector* supersymmetric quantum mechanics

$$H_{\rm BPS} = \frac{1}{2}\sum_A \{Q_A, Q_A^\dagger\} = -\frac{1}{2}\nabla^2 + \frac{1}{2}(\nabla W)^2 + \frac{1}{2}\frac{\partial^2 W}{\partial x^i \partial x^j}[\psi^{i\dagger}, \psi^j] \tag{24}$$

Because of mutual force balance between BPS particles $W \approx 0$, hence, of ideal gas type.

It is now clear how macroscopically charged D-particle behave differently from field theory BPS-particle. At classical level D-particles behave indifferently from BPS particles: both experience no net force because of BPS nature. At quantum level, however, D-particles experience logarithmically repulsive quantum effective potential. Because of this quantum pressure, average spacing between constituent D-particles is of string scale $\mathcal{O}(\sqrt{\alpha'})$. Collective excitation of D-particle gas is that of one-dimensional Bose gas described by two-dimensional free scalar field theory. For field theory D-particles, no quantum effective potential , hence, no quantum pressure is generated. So long as intrinsic size is ignored these BPS particles can overlap freely.

The above argument is not restricted to D-particles but extends to other D-branes. For instance, consider 8-branes in type IIB superstring theory. If compactified on a circle and worldsheet orbifoldized, one obtains type-I' string. Uniformly weak coupling configuration is when two sets of 16 8-branes are located at the $\mathbf{Z}_2$ fixed points $X^9 = 0, 2\pi/R_I$. While 16 is not a terribly large number, let us pretend so and study many-body aspects. Their low-energy excitation is given by the transverse locations of 8-brane center of masses. Again this is described by $N = 8$ supersymmetric $O(16)$ *matrix* quantum mechanics projected to a gauge singlet sector. At quantum level, equilibrium positions are when the inter-spacing of elementary 8-branes is of order $\sqrt{\alpha'}$.

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REFERENCES

[1] C. Hull and P. Townsend, *Nucl. Phys.* B **438**, 109 (1995); E. Witten, *Nucl. Phys.* B **443**, 85 (1995).

[2] A. Dabholkar and J.A. Harvey, *Phys. Rev. Lett.* **63**, 478 (1989).

[3] S.-J. Rey, *Axionic String Instantons & Their Low-Energy Implications,* in *Superstring and Particle Theory*, eds. L Clavelli, B Harms, p.291-300 (World Scientific, Singapore, 1990); *On String Theory and Axionic Strings & Instantons,* in *Particles and Fields '91*, eds. D Axon, D Bryman, M Comyn, p.876-881 (World Scientific, Singapore, 1992).

[4] S.-J. Rey, *Phys. Rev.* D **43**, 526 (1991).

[5] A. Strominger, *Nucl. Phys.* B **343**, 167 (1990), Erratum **353**, 565 (1991).

[6] C.G. Callan, J.A. Harvey and A. Strominger, *Nucl. Phys.* B **359**, 611 (1991); *Nucl. Phys.* B **367**, 60 (1991).

[7] For a recent review, see M.J. Duff, R.R. Khuri and J.X. Lu, *Phys. Rep.* **259**, 213 (1995).

[8] C.G. Callan, J.A. Harvey and A. Strominger, *Supersymmetric String Solitons,* in *String Theory and Quantum Gravity '91*, eds. , p.208-244 (World Scientific, Singapore, 1992).

[9] J. Polchinski, *Phys. Rev. Lett.* **75**, 4724 (1996).

[10] S.-J. Rey, *Collective Coordinate Dynamics and Semiclassical Quantization of Dirichlet Brane,* SNUTP 96/014 preprint (1996).

[11] E. Cremmer and J. Scherk, *Nucl. Phys.* B **50**, 222 (1972).

[12] E. Witten, *Nucl. Phys.* B **460**, 335 (1996).

[13] S.-J. Rey, *Phys. Lett.* B **203**, 393 (1988); J. Liu and J. Polchinksi, *Phys. Lett.* B **203**, 39 (1988).

[14] J. Polchinski, *Phys. Rev.* D **50**, R6041 (1994).

[15] M.B. Green, *Phys. Lett.* B **69**, 89 (1977); *Phys. Lett.* B **201**, 42 (1988); *Phys. Lett.* B **282**, 380 (1992); M.B. Green and J. Polchinski, *Phys. Lett.* B **335**, 377 (1994).

[16] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, *Nucl. Phys.* B **308**, 221 (1988).

[17] W. Fischler, S. Paban and M. Rozali, *Phys. Lett.* B **352**, 298 (1995).

[18] R.P. Feynman, *Phys. Rep.* **80**, 440 (1950); Y. Nambu, *Prog. Theo. Phys.* **5**, 82 (1950). See also B. Sathiapalan, *Nucl. Phys.* B **294**, 747 (1987).

[19] W. Fischler and L. Susskind, *Phys. Lett.* B **171**, 383 (1986); *Phys. Lett.* B **173**, 262 (1986).

[20] S.R. Das and S.-J. Rey, *Phys. Lett.* B **186**, 328 (1987).

[21] C. Lovelace, *Nucl. Phys.* B **273**, 413 (1986); T. Bank and E. Martinec, *Nucl. Phys.* B **294**, 733 (1987); J. Hughes, J. Liu and J. Polchinski, *Nucl. Phys.* B **316**, 15 (1989).

[22] S.R. Das, *Phys. Rev.* D **38**, 3105 (1988).

[23] E. Witten, *Phys. Rev.* D **46**, 5467 (1992); E. Verlinde, *Nucl. Phys.* B **377**, 141 (1992).

[24] S.R. Das, S. Naik and S.R. Wadia, *Mod. Phys. Lett.* A **4**, 1033 (1989).

[25] A. Cooper, L. Susskind and L. Thorlacius, *Nucl. Phys.* B **363**, 132 (1991).